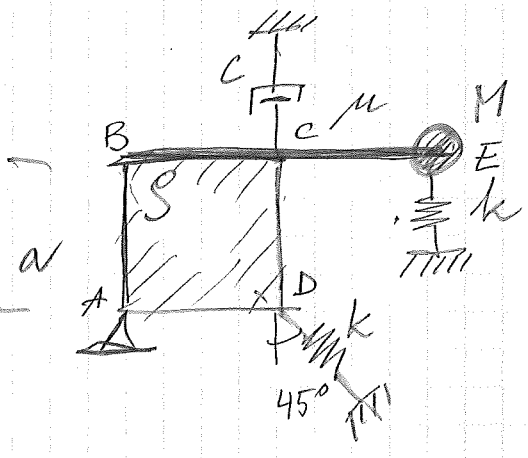


$\varrho = 50\ kg/m^2$

$\mu = 60\ kg/m$

$k = 10^6\ N/m$

$c = 10\ kNs/m$

$a = 3m$

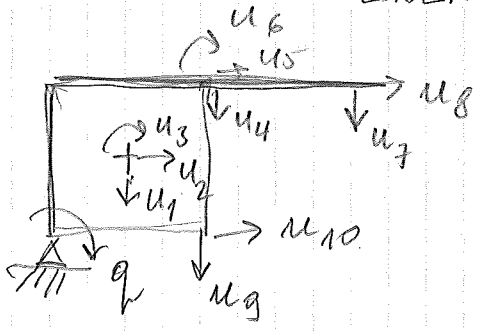
$M = \mu \cdot a = 180\ kg$

| a | a |

I Współrzędna q → obrót w p. A

ENERGIA KINETYCZNA

$E_k = E_k(\varrho) + E_k(\mu) + E_k(M)$



$u_3 = u_6 = q$

$u_1 = q \cdot a/2$

$u_2 = q \cdot a/2$

$u_4 = q \cdot a$

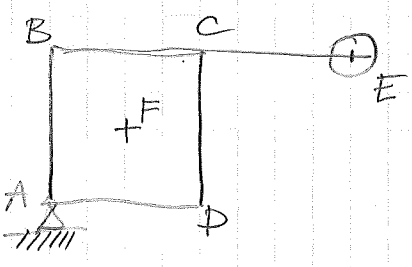
$u_5 = q \cdot a$

$u_7 = q \cdot 2a$

$u_8 = q \cdot a$

$u_9 = qa$

$u_{10} = 0$



$E_k = E_k(M) + E_k(\varrho) + E_k(\mu)$

$E_k(M) = \frac{1}{2} M \dot{u}_7^2 + \frac{1}{2} M \dot{u}_8^2 =$

$= \frac{1}{2} M\, 4a^2 \dot{q}^2 + \frac{1}{2} M a^2 \dot{q}^2 = \frac{1}{2}(5Ma^2)\dot{q}^2$

т. "F"

$$E_k(s) = \frac{1}{2} m_T \dot{u}_1^2 + \frac{1}{2} m_T \dot{u}_2^2 + \frac{1}{2} J_{TF} \cdot \dot{u}_3^2 =$$

$$= \frac{1}{2} \rho a^2 \left(\dot{q} \frac{a}{2}\right)^2 + \frac{1}{2} \rho a^2 \left(\dot{q} \frac{a}{2}\right)^2 + \frac{1}{2} \left[\rho \left(\frac{1}{12} a \cdot a^3 + \right.\right.$$

$$\left.\left. + \frac{1}{12} a \cdot a^3\right)\right] \cdot \dot{q}^2 =$$

$$= \frac{1}{2} \cdot \frac{1}{4} \rho a^4 \dot{q}^2 + \frac{1}{2} \cdot \frac{1}{4} \rho a^4 \dot{q}^2 + \frac{1}{2} \cdot \frac{1}{6} \rho a^4 \dot{q}^2 =$$

$$= \frac{1}{2} \cdot \frac{2}{3} \rho a^4 \dot{q}^2$$

т. "B"

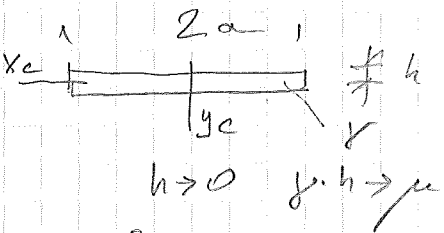
$$E_k(s) = \frac{1}{2} J_{TA} \dot{u}_3^2 = \frac{1}{2} \left[\rho \left(\frac{1}{3} a \cdot a^3 + \frac{1}{3} a \cdot a^3\right)\right] \dot{q}^2 =$$

$$= \frac{1}{2} \cdot \frac{2}{3} \rho a^4 \dot{q}^2$$

$E_k(\mu)$ p. C

$$E_k(\mu) = \frac{1}{2} m_p \dot{u}_4^2 + \frac{1}{2} m_p \dot{u}_5^2 + \frac{1}{2} J_{pc} \dot{u}_6^2$$

$$m_p = 2a \cdot \mu$$



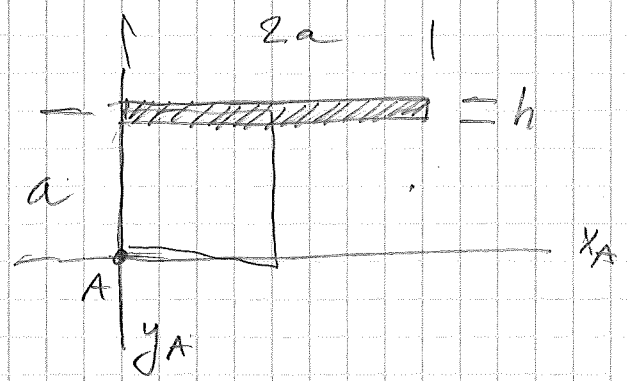
$$J_{pc} = \gamma \left(\frac{1}{12} \cdot 2a \cdot h^3 + \frac{1}{12} \cdot h \cdot (2a)^3\right) =$$

$$= \frac{1}{12} \cdot 2a \cdot (\gamma h \to \mu)\, h^2 \to 0 + \frac{1}{12} (\gamma h \to \mu) \cdot 8a^3 = \frac{2}{3} \mu a^3$$

$$E_k(\mu) = \frac{1}{2} 2a\mu \cdot (\dot{q}a)^2 + \frac{1}{2} 2a\mu (\dot{q}a)^2 +$$

$$+ \frac{1}{2} \cdot \frac{2}{3} \mu a^3 \cdot \dot{q}^2 = \frac{1}{2} \cdot \frac{14}{3} \mu a^3 \dot{q}^2$$

p. A



$h \to 0 \qquad \gamma h \to \mu$

$$J_{pA} = \gamma (J_{xA} + J_{yA})$$

$$J_{xA} = \frac{1}{12} \cdot 2a \cdot h^3 + 2a \cdot h \cdot a^2$$

$$J_{yA} = \frac{1}{3} h \cdot (2a)^3$$

$$J_{pA} = \frac{1}{12} 2a \cdot \gamma h \cdot h^3 + 2a \cdot \gamma h \cdot a^2 + \frac{1}{3} \gamma h \cdot 8a^3$$

($\frac{1}{12} 2a \cdot \gamma h \cdot h^3 \to 0$)

$$= 2\mu a^3 + \frac{8}{3}\mu a^3 = \frac{14}{3}\mu a^3$$

$$E_k(pa) = \frac{1}{2} J_{pA} \cdot \dot{\varphi}_3^2 = \frac{1}{2} \frac{14}{3} \mu a^3 \dot{q}^2$$

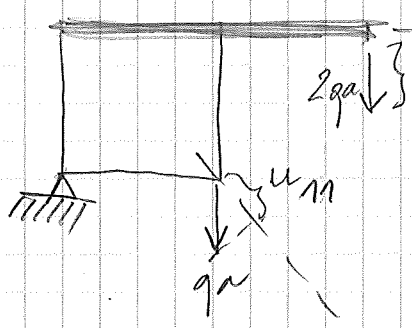
$$E_k = \frac{1}{2} 5Ma^2 \dot{q}^2 + \frac{1}{2} \frac{2}{3} \rho a^4 \dot{q}^2 + \frac{1}{2} \frac{14}{3} \mu a^3 \dot{q}^2$$

$$= \frac{1}{2} \left(5Ma^2 + \frac{2}{3}\rho a^4 + \frac{14}{3}\mu a^3\right) \dot{q}^2$$

$$m_{zast} = 5Ma^2 + \frac{2}{3}\rho a^4 + \frac{14}{3}\mu a^3 = 8,100 + 2700 +$$

$$+ 7560 = 18360 \ kgm^2$$

ENERGIA POTENCJALNA



$$E_p = \frac{1}{2} k \cdot u_{11}^2 + \frac{1}{2} k \cdot u_7^2$$

$$u_{11} = qa \cdot \frac{\sqrt{2}}{2} \qquad u_7 = 2qa$$

$$E_p = \frac{1}{2} k \left(qa \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2} k (2qa)^2$$

$$= \frac{1}{2} k \frac{1}{2} q^2 a^2 + \frac{1}{2} k 4 q^2 a^2 =$$

$$= \frac{1}{2} \left(\frac{1}{2} k a^2 + 4 k a^2\right) q^2 =$$

$$= \frac{1}{2} 4,5 k a^2 q^2 = \frac{1}{2} k_{zast} q^2$$

$k_{zast} = 4,5 k a^2 = 40,5 \cdot 10^6 \ \frac{N}{m} \cdot m^2 = \frac{Nm}{rad}$

MOC TŁUMIENIA

$$\Phi = \frac{1}{2} c \cdot \dot{u}_4^2 = \frac{1}{2} c (\dot{\varphi} a)^2 = \frac{1}{2} c a^2 \dot{\varphi}^2$$

$$\Phi = \frac{1}{2} c_{zast} \dot{\varphi}^2$$

$$c_{zast} = ca^2 = 10 \cdot 10^3 \cdot 3^2 = 90 \cdot 10^3 \left[\frac{Ns}{m} \cdot m^2 = \frac{Nms}{rad} \right]$$

CZĘSTOŚĆ DRGAŃ WŁASNYCH

$$\omega = \sqrt{\frac{k_{zast}}{m_{zast}}} = \sqrt{\frac{40{,}5 \cdot 10^6}{18360}} = 46{,}97 \ \frac{rad}{s}$$

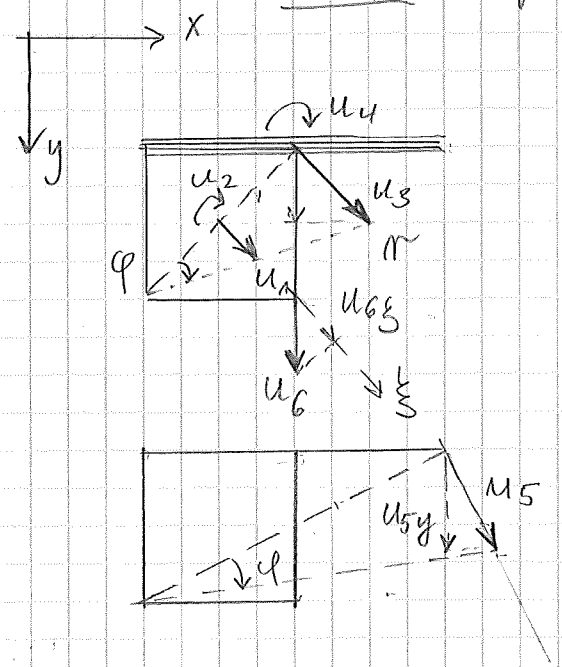
UŁAMEK TŁUMIENIA KRYTYCZNEGO

$$\alpha = \frac{c_{zast}}{2\sqrt{k_{zast} m_{zast}}} = \frac{90 \cdot 10^3}{2\sqrt{40{,}5 \cdot 10^6 \cdot 18360}} =$$

$$= 0{,}052 = 5{,}2\%$$

II Współrzędna u → przemieszczenie (całkowite) p. C

UWAGA: współrzędne lokalne „u" mają inne znaczenie niż w p. I



$$\varphi \equiv \operatorname{tg}\varphi = \frac{r}{a\sqrt{2}}$$

$$u_1 = \varphi \cdot a \frac{\sqrt{2}}{2} = \frac{1}{2} r$$

$$u_2 = u_4 = \varphi = r / a\sqrt{2} \ ; \ u_3 = r$$

$$u_5 = \varphi \cdot \sqrt{a^2 + (2a)^2} = \varphi \cdot a\sqrt{5}$$

$$= \frac{r}{a\sqrt{2}} \cdot a\sqrt{5} = r\sqrt{\frac{5}{2}}$$

$$u_6 = \varphi \cdot a = \frac{r}{\sqrt{2}} = r\frac{\sqrt{2}}{2}$$

ENERGIA KINETYCZNA

$E_k = E_k(\varsigma) + E_k(\mu) + E_k(M)$

- tarcza masowa

p. „F"

przemieszczenie u_1 opisuje całkowite przemieszczenie środka tarczy

$E_k(\varsigma) = \frac{1}{2} m_T \cdot \dot{u}_1^2 + \frac{1}{2} J_{TF} \cdot \dot{u}_2^2$

$m_T = \varsigma a^2 \qquad J_{TF} = \frac{1}{6}\varsigma a^4$ (p. str. 2)

$E_k(\varsigma) = \frac{1}{2}\varsigma a^2 \left(\frac{1}{2}\dot{r}\right)^2 + \frac{1}{2}\,\frac{1}{6}\varsigma a^4 \left(\frac{\dot{r}}{a\sqrt{2}}\right)^2 =$

$= \frac{1}{2}\varsigma a^2 \frac{1}{4}\dot{r}^2 + \frac{1}{2}\frac{1}{6}\varsigma a^4 \frac{\dot{r}^2}{2a^2} = \frac{1}{2}\left(\frac{1}{4}\varsigma a^2 + \frac{1}{12}\varsigma a^2\right)\dot{r}^2 =$

$= \frac{1}{2}\,\frac{1}{3}\varsigma a^2 \dot{r}^2$

p. „A"

$E_k(\varsigma) = \frac{1}{2} J_{TA} \cdot \dot{\varphi}^2 \qquad J_{TA} = \frac{2}{3}\varsigma a^4$ (p. str. 2)

$E_k(\varsigma) = \frac{1}{2}\,\frac{2}{3}\varsigma a^4 \left(\frac{\dot{r}}{a\sqrt{2}}\right)^2 = \frac{1}{2}\,\frac{1}{3}\varsigma a^2 \dot{r}^2$

- punkt masowy

$E_k(M) = \frac{1}{2} M \cdot \dot{u}_5^2 = \frac{1}{2} M \left(\dot{r}\sqrt{\frac{5}{2}}\right)^2 =$

$= \frac{1}{2}\,\frac{5}{2} M \dot{r}^2$

- pręt masowy

|| p. „C" $\qquad J_{pc} = \frac{2}{3}\mu a^3$ (p. str. 2)

$E_k(\mu) = \frac{1}{2} m_p \cdot \dot{u}_3^2 + \frac{1}{2} J_{pc} \dot{u}_2^2 = \frac{1}{2}\, 2a\mu \cdot \dot{r}^2 +$

$+ \frac{1}{2}\,\frac{2}{3}\mu a^3 \left(\frac{\dot{r}}{a\sqrt{2}}\right)^2 = \frac{1}{2}\left(2a\mu + \frac{1}{3}a\mu\right)\dot{r}^2 =$

$= \frac{1}{2} \cdot \frac{7}{3}\mu a \dot{r}^2$

p „A" $J_{pA} = \frac{14}{3} \mu a^3$ (p. str 3)

$E_k(\mu) = \frac{1}{2} J_{pA} \cdot \dot{u}_4^2 = \frac{1}{2} \cdot \frac{14}{3} \mu a^3 \left(\frac{\dot{r}}{a\sqrt{2}}\right)^2 =$

$= \frac{1}{2} \frac{7}{3} \mu a \dot{r}^2$

$E_k = E_k(\varrho) + E_k(\mu) + E_k(M) = \frac{1}{2}\left(\frac{1}{3}\varrho a^2 + \frac{7}{3}\mu a + \frac{5}{2}M\right)\dot{r}^2$

$m_{zast} = \frac{1}{3}\varrho a^2 + \frac{7}{3}\mu a + \frac{5}{2}M = 160 + 420 + 450 = 1020\ kg$

ENERGIA POTENCJALNA

$E_p = \frac{1}{2} k \cdot u_{6\xi}^2 + \frac{1}{2} k \cdot u_{5y}^2$

$u_{6\xi} = u_6 \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} r$ $u_{5y} = u_5 \cdot \frac{2}{\sqrt{2^2+1^2}} = u_5 \cdot \frac{2}{\sqrt{5}} = r\sqrt{2}$

(albo $u_{5y} = \varphi \cdot 2a = r\sqrt{2}$)

$E_p = \frac{1}{2} k \left(\frac{1}{2} r\right)^2 + \frac{1}{2} k (r\sqrt{2})^2 = \frac{1}{2}\left(\frac{1}{4}k + 2k\right) r^2 = \frac{1}{2} \frac{9}{4} k r^2$

$k_{zast} = \frac{9}{4} k = 2{,}25 \cdot 10^6\ N/m$

MOC TŁUMIENIA

$\Phi = \frac{1}{2} c \cdot \dot{u}_{3y}^2$ $u_{3y} = r \cdot \frac{\sqrt{2}}{2}$

$\Phi = \frac{1}{2} c \left(\dot{r} \frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} \frac{1}{2} c \dot{r}^2$

$c_{zast} = \frac{1}{2} c = 5 \cdot 10^3\ \frac{Ns}{m}$

CZĘSTOŚĆ DRGAŃ WŁASNYCH

$\omega = \sqrt{\frac{k_{zast}}{m_{zast}}} = \sqrt{\frac{2{,}25 \cdot 10^6}{1020}} = 46{,}97\ rad/s$

UŁAMEK TŁUMIENIA KRYTYCZNEGO

$\alpha = \frac{c_{zast}}{2\sqrt{k_{zast} m_{zast}}} = \frac{5 \cdot 10^3}{2\sqrt{2{,}25 \cdot 10^6 \cdot 1020}}$

$\alpha = 0{,}052 = 5{,}2\ \%$