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THE DEVELOPMENT OF THE POTENTIAL AND ACADEMIC PROGRAMMES OF WROCŁAW UNIVERSITY OF TECHNOLOGY

Dynamics of Structures

Part 2

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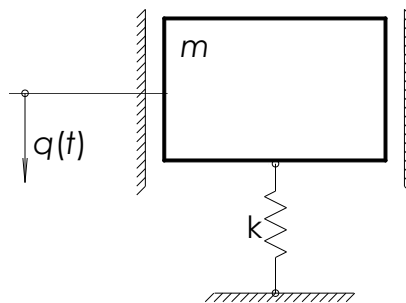


Single-degree-of-freedom systems

Undamped systems

The equation of motion for an undamped single-degree-of-freedom system can be written in form

$$m\ddot{q} + kq = 0$$



This is the second order differential equation with constant coefficients. The solution of this equation can be found with the help of the system “*Mathematica*”. The variable “eqofmotion1” contains the equation of motion without damping (In[1]), “solut1” contains the solution of this equation with initial conditions for time $t=0$ (initial displacement – q_0 , initial velocity – $\dot{q}_0$). The symbol “ ’ ” denotes a derivative, so you can plot displacement $q[t]$, velocity $q'[t]$ (first derivative), and acceleration $q''[t]$ (second derivative). The new function $q1[t]$ is defined (In[4]).

```
In[1]:= eqofmotion1 = m * q''[t] + k * q[t] == 0
```

```
Out[1]= k q[t] + m q''[t] == 0
```

```
In[2]:= solut1 = DSolve[{eqofmotion1, q[0] == q0, q'[0] == qp0}, q[t], t]
```

```
Out[2]= {{q[t] -> \frac{\sqrt{k} q_0 \cos\left[\frac{\sqrt{k} t}{\sqrt{m}}\right] + \sqrt{m} q_{p0} \sin\left[\frac{\sqrt{k} t}{\sqrt{m}}\right]}{\sqrt{k}}}}
```



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```
In[3]:= solut1 // TraditionalForm
```

```
Out[3]//TraditionalForm=
```

$$\left\{ \left\{ q(t) \rightarrow \frac{\sqrt{k} q_0 \cos\left(\frac{\sqrt{k} t}{\sqrt{m}}\right) + \sqrt{m} q_{p0} \sin\left(\frac{\sqrt{k} t}{\sqrt{m}}\right)}{\sqrt{k}} \right\} \right\}$$

```
In[4]:= q1[t_] = solut1[[1, 1, 2]]
```

```
Out[4]=
```

$$\frac{\sqrt{k} q_0 \cos\left[\frac{\sqrt{k} t}{\sqrt{m}}\right] + \sqrt{m} q_{p0} \sin\left[\frac{\sqrt{k} t}{\sqrt{m}}\right]}{\sqrt{k}}$$

By evaluating $q1[0]$ and $q1'[0]$, you obtain the initial conditions:

```
q1[0]
```

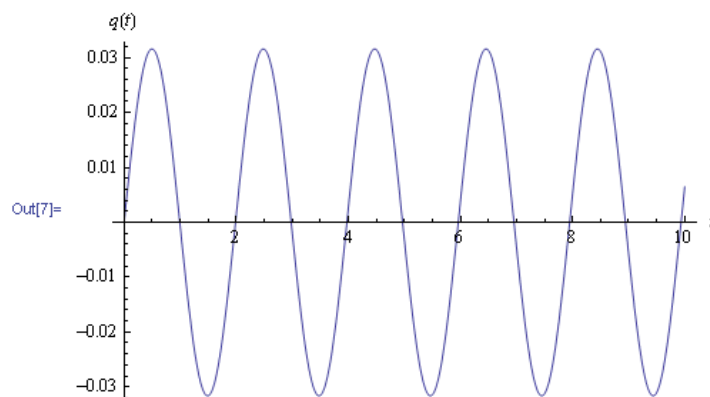
```
Out[5]= q0
```

```
q1'[0]
```

```
Out[6]= qp0
```

The particle solution can be obtained from the general solution by determining the initial conditions “ q_0 ” “ q_{p0} ”. The command `Block` in “*Mathematica*” makes it possible to achieve the particular solution for the values: $m=1000$ kg, $k=10000$ N/m, $q_0=0$ m, $q_{p0}=0,1$ m/s.

```
In[7]:= Block[{m = 1000, k = 10 000, q0 = 0, qp0 = 0.1}, Plot[q1[t], {t, 0, 10}, AxesLabel -> {t, q[t]}]]
```

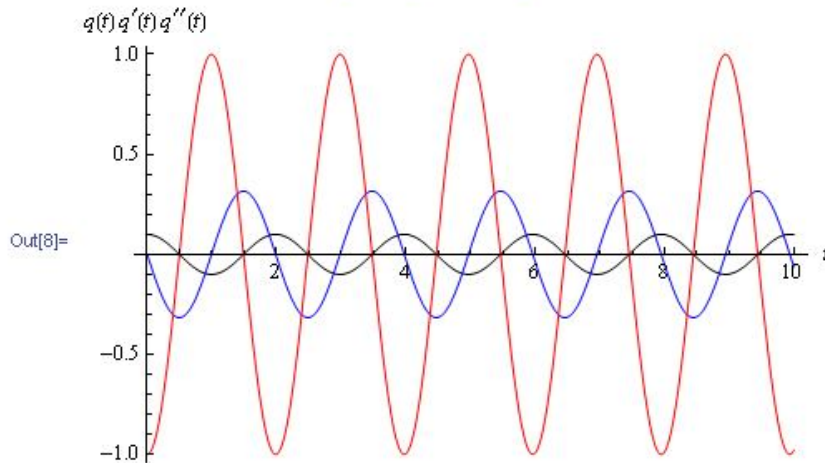


Using the option `PlotStyle`, you can change the properties of the plot. In the example below, the displacement is plotted in black, the velocity – in blue and the acceleration – in red.



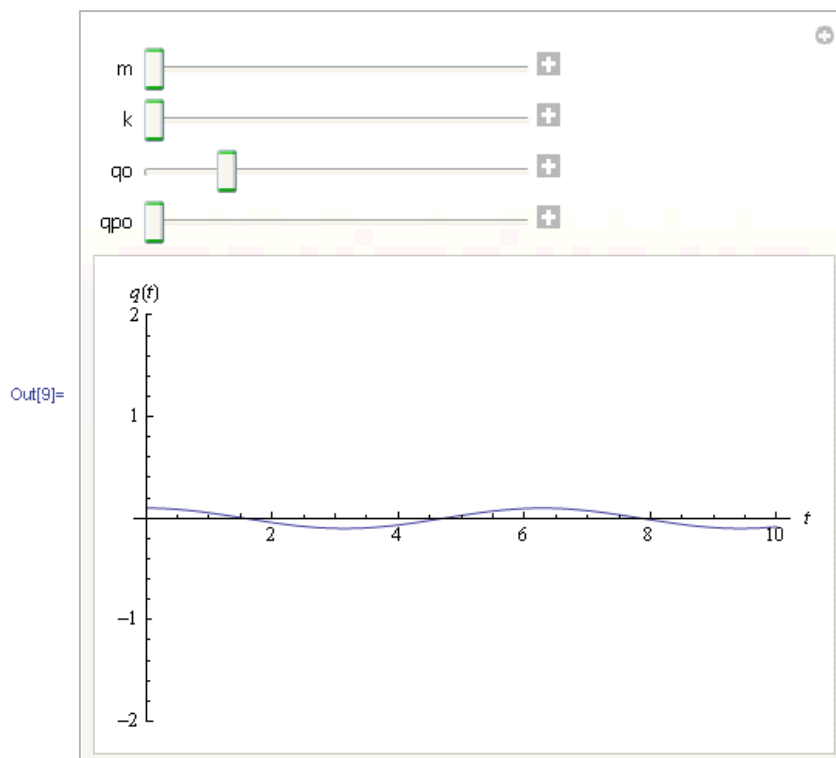
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```
In[8]:= Block[{m = 1000, k = 10 000, q0 = 0.1, qpo = 0},
  Plot[{q1[t], q1'[t], q1''[t]}, {t, 0, 10}, PlotStyle -> {Black, Blue, Red},
  AxesLabel -> {t, q[t] q'[t] q''[t]}]
```



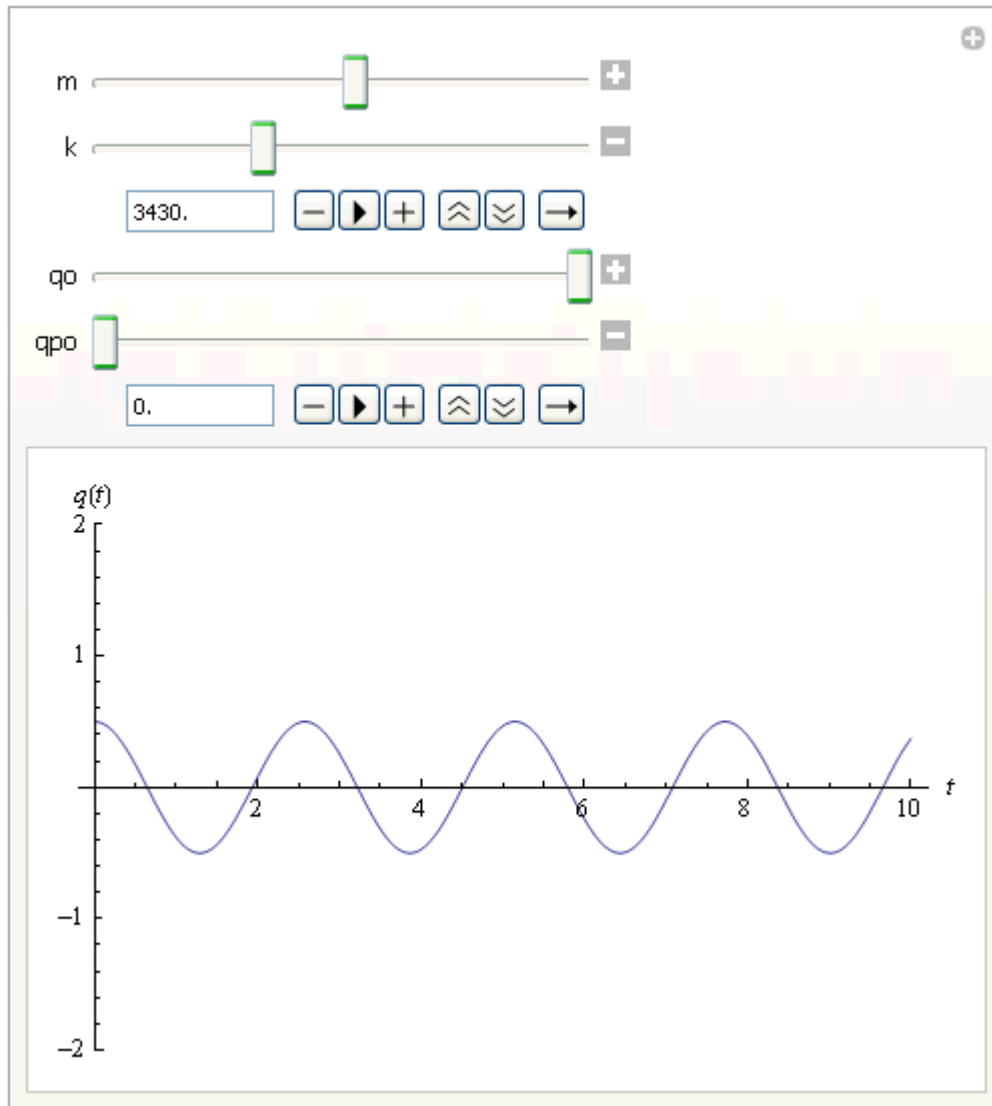
By using the command `Manipulate` with the command `Plot` nested, you can observe how the solution changes with changing parameters. Command `Plot` are followed by parameters (here m , k , q_0 , q_{po}) with ranges and optionally initial values.

```
In[9]:= Manipulate[Plot[1/sqrt[k] (sqrt[k] q0 Cos[sqrt[k] t / sqrt[m]] + sqrt[m] qpo Sin[sqrt[k] t / sqrt[m]]), {t, 0, 10}, PlotRange -> {-2, 2},
  AxesLabel -> {t, q[t]}], {m, 100, 1000}, {k, 100, 10 000}, {{q0, 0.1}, 0, 0.5}, {qpo, 0, 0.5}]
```



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By moving the sliders to the left or right, you can observe changes in the plot. By pushing the “+” button, you activate the sub-menu

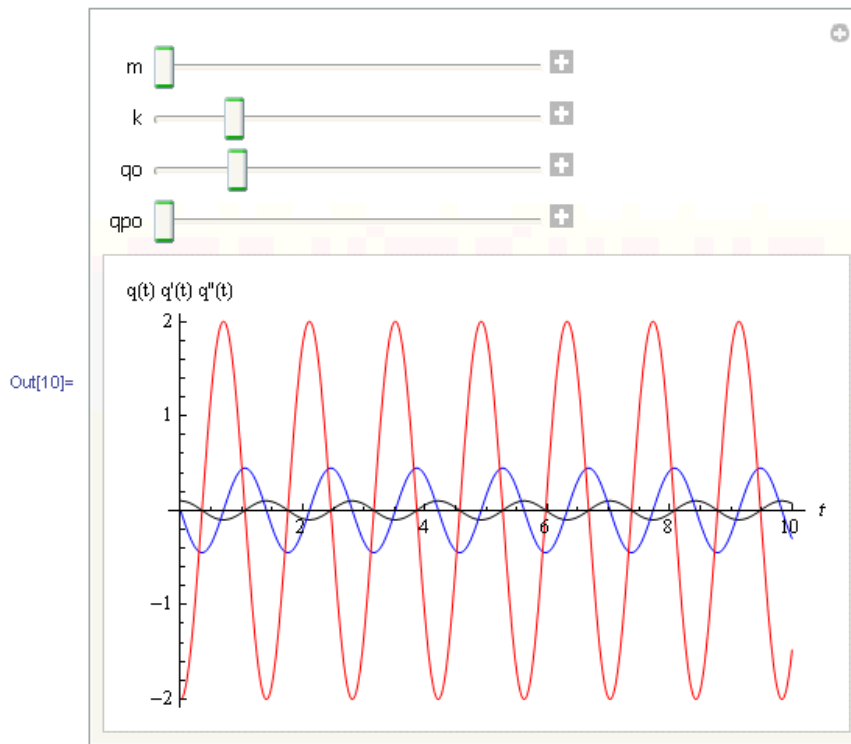


You can also plot displacement, velocity and acceleration by applying the command `Manipulate`

```
In[10]:= Manipulate[ temp[t_] := 
$$\frac{\sqrt{k} q_0 \cos\left[\frac{\sqrt{k} t}{\sqrt{m}}\right] + \sqrt{m} q_{p0} \sin\left[\frac{\sqrt{k} t}{\sqrt{m}}\right]}{\sqrt{k}};$$

Plot[{temp[t], temp'[t], temp''[t]}, {t, 0, 10}, PlotStyle -> {Black, Blue, Red},
AxesLabel -> {t, "q(t) q'(t) q''(t)"}, {m, 100, 1000}, {{k, 2000}, 100, 10000},
{{q0, 0.1}, 0, 0.5}, {qp0, 0, 0.5}]
```

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Task:

Find which parameters decrease the dynamic response, and which ones increase it.

Damped system

The equation of motion for a damped single-degree-of-freedom system can be written in form

$$m\ddot{q} + c\dot{q} + kq = 0$$

$$\text{eqofmotion2} = m * q''[t] + c * q'[t] + k * q[t] == 0$$

$$k q[t] + c q'[t] + m q''[t] == 0$$

$$\text{solut2} = \text{DSolve}[\{\text{eqofmotion2}, q[0] == q_0, q'[0] == qp_0\}, q[t], t]$$

$$\left\{ \left\{ q[t] \rightarrow \frac{1}{2\sqrt{c^2 - 4km}} \left(-c e^{\frac{(-c - \sqrt{c^2 - 4km})t}{2m}} q_0 + c e^{\frac{(-c + \sqrt{c^2 - 4km})t}{2m}} q_0 + e^{\frac{(-c - \sqrt{c^2 - 4km})t}{2m}} \sqrt{c^2 - 4km} q_0 + e^{\frac{(-c + \sqrt{c^2 - 4km})t}{2m}} \sqrt{c^2 - 4km} q_0 - 2 e^{\frac{(-c - \sqrt{c^2 - 4km})t}{2m}} m qp_0 + 2 e^{\frac{(-c + \sqrt{c^2 - 4km})t}{2m}} m qp_0 \right) \right\} \right\}$$



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FullSimplify[solut2]

$$\left\{ \left\{ q[t] \rightarrow \frac{e^{-\frac{c}{2m}t} \left(\sqrt{c^2 - 4km} q_0 \cosh\left[\frac{\sqrt{c^2 - 4km}t}{2m}\right] + (c q_0 + 2m q_{p0}) \sinh\left[\frac{\sqrt{c^2 - 4km}t}{2m}\right] \right)}{\sqrt{c^2 - 4km}} \right\} \right\}$$

q2[t_] = solut2[[1, 1, 2]]

$$\frac{1}{2\sqrt{c^2 - 4km}} \left(-c e^{\frac{(-c - \sqrt{c^2 - 4km})t}{2m}} q_0 + c e^{\frac{(-c + \sqrt{c^2 - 4km})t}{2m}} q_0 + e^{\frac{(-c - \sqrt{c^2 - 4km})t}{2m}} \sqrt{c^2 - 4km} q_0 + \right. \\ \left. e^{\frac{(-c + \sqrt{c^2 - 4km})t}{2m}} \sqrt{c^2 - 4km} q_0 - 2 e^{\frac{(-c - \sqrt{c^2 - 4km})t}{2m}} m q_{p0} + 2 e^{\frac{(-c + \sqrt{c^2 - 4km})t}{2m}} m q_{p0} \right)$$

q2[0]

q₀

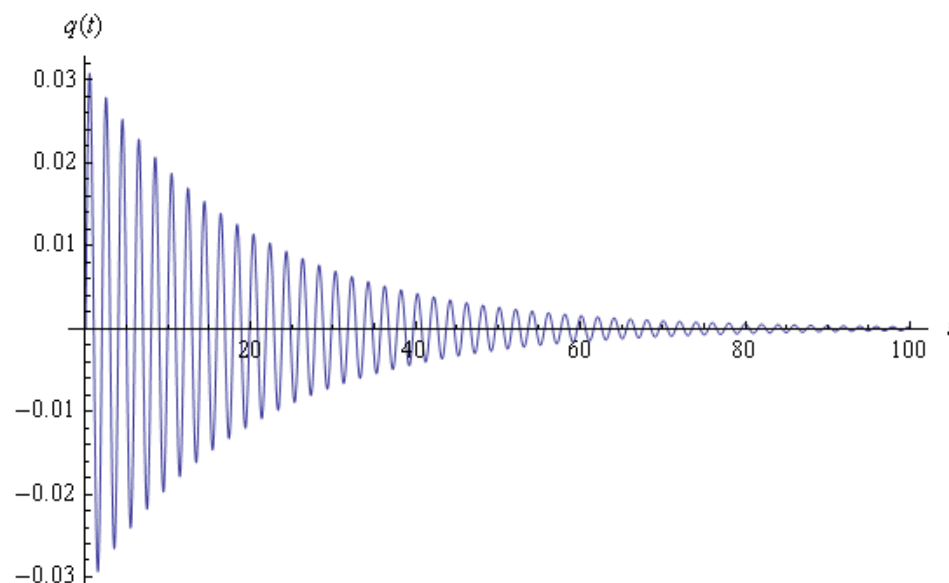
q2'[0]

$$\frac{1}{2\sqrt{c^2 - 4km}} \left(-\frac{c(-c - \sqrt{c^2 - 4km})}{2m} q_0 + \frac{\sqrt{c^2 - 4km}(-c - \sqrt{c^2 - 4km})}{2m} q_0 + \frac{c(-c + \sqrt{c^2 - 4km})}{2m} q_0 + \right. \\ \left. \frac{\sqrt{c^2 - 4km}(-c + \sqrt{c^2 - 4km})}{2m} q_0 - (-c - \sqrt{c^2 - 4km}) q_{p0} + (-c + \sqrt{c^2 - 4km}) q_{p0} \right)$$

FullSimplify[q2'[0]]

q_{p0}

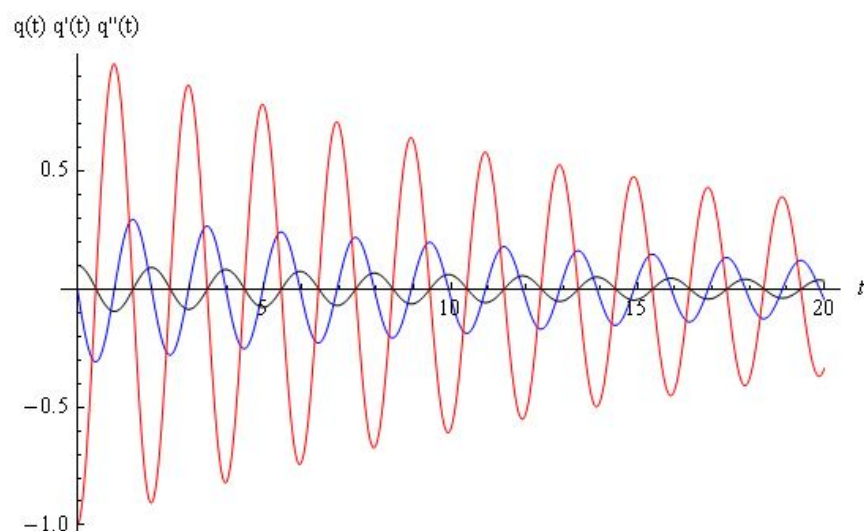
**Block[{m = 1000, k = 10000, c = 100, q₀ = 0, q_{p0} = 0.1},
Plot[q2[t], {t, 0, 100}, PlotRange → All, AxesLabel → {t, q[t]}]]**





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```
Block[{m = 1000, k = 10 000, c = 100, q0 = 0.1, qp0 = 0},
Plot[{q2[t], q2'[t], q2''[t]}, {t, 0, 20},
AxesLabel -> {t, "q(t) q'(t) q''(t)"}, PlotStyle -> {Black, Blue, Red}]]
```

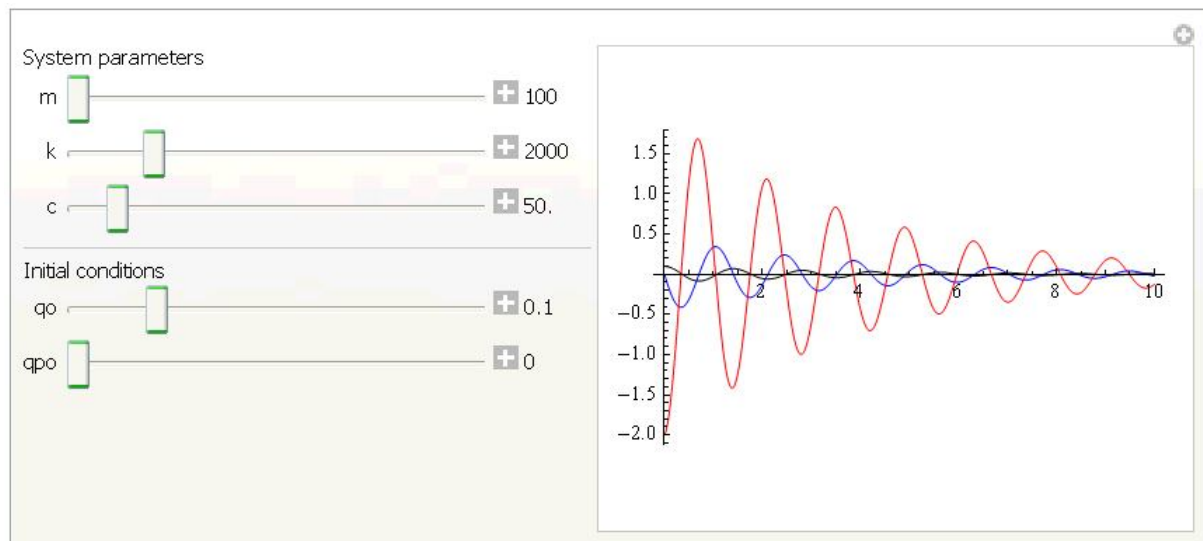


```
Manipulate[
temp[t_] :=

$$\frac{e^{-\frac{c}{2m}t} \left( \sqrt{c^2 - 4km} q_0 \cosh\left[\frac{\sqrt{c^2 - 4km}t}{2m}\right] + (c q_0 + 2m qp_0) \sinh\left[\frac{\sqrt{c^2 - 4km}t}{2m}\right] \right)}{\sqrt{c^2 - 4km}};$$

Plot[{temp[t], temp'[t], temp''[t]}, {t, 0, 10}, PlotRange -> All,
PlotStyle -> {Black, Blue, Red}], AxesLabel -> {t, "q(t) q'(t) q''(t)"},
"System parameters", {m, 100, 1000, Appearance -> "Labeled"},
{{k, 2000}, 100, 10 000, Appearance -> "Labeled"},
{c, 0, 500, Appearance -> "Labeled"}, Delimiter, "Initial conditions",
{{q0, 0.1}, 0, 0.5, Appearance -> "Labeled"},
{qp0, 0, 0.5, Appearance -> "Labeled"}, ControlPlacement -> Left]
```

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Task:

- Find which parameters decrease the dynamic response, and which ones increase it.
- Find which parameters decrease the vibration period, and which ones increase it

Harmonically forced vibration

The equation of motion for forced vibration of a single-degree-of-freedom system (SDOF) can be written in form

$$m\ddot{q} + c\dot{q} + kq = F(t)$$

The vibration occurring when the excitation force is equal to $F(t) = F_s \sin pt + F_c \cos pt$ is called harmonically forced vibration.

The equation of motion for a system with one degree of freedom is declared as the variable `eqofmotion3`. The system is harmonically excited with the angular frequency p .

```
% eqofmotion3 = m * q''[t] + c * q'[t] + k * q[t] = F0 * Sin[p * t]
% k * q[t] + c * q'[t] + m * q''[t] = F0 Sin[p * t]
```

This equation has been solved using the command `DSolve`. The initial condition was taken as equal to zero.



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```
solut3a = DSolve[{eqofmotion3, q[0] == 0, q'[0] == 0}, q[t], t];
```

```
FullSimplify[solut3a] // TraditionalForm
```

$$\left\{ \left\{ q(t) \rightarrow \left(e^{-\frac{(c+\sqrt{c^2-4km})t}{2m}} F_0 \left(p \left(\left(-1 + e^{\frac{\sqrt{c^2-4km}t}{m}} \right) c^2 + \left(1 + e^{\frac{\sqrt{c^2-4km}t}{m}} \right) \sqrt{c^2-4km} c + 2 \left(-1 + e^{\frac{\sqrt{c^2-4km}t}{m}} \right) m(m p^2 - k) \right) - \right. \right. \right. \right. \\ \left. \left. \left. 2 e^{\frac{(c+\sqrt{c^2-4km})t}{2m}} \sqrt{c^2-4km} (c p \cos(pt) + (m p^2 - k) \sin(pt)) \right) \right) \right) / \left(2 \sqrt{c^2-4km} (c^2 p^2 + (k - m p^2)^2) \right) \right\} \right\}$$

Function $q3a[t]$ was declared as the solution $solut3a$ of the equation of motion $eqofmotion3$. The initial conditions are checked below.

```
q3a[t_] = solut3a[[1, 1, 2]];
```

```
q3a[0]
```

```
0
```

```
Simplify[q3a'[0]]
```

```
0
```

The figure below shows the first 100 s of vibration of an SDOF system. The SDOF system has the parameters: $m = 1000$ kg, $c = 80$ Ns/m, $k = 9000$ N/m, $F_s = 10000$ N = 10 kN, $p = 20$ rad/s. The natural angular frequency for this system equals

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{9000}{1000}} = 3,0 \text{ rad/s}$$

the natural frequency is

$$f_n = \frac{\omega_n}{2\pi} = \frac{17,68}{2\pi} = 0,477 \text{ Hz}$$

the period of undamped free vibrations is

$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} = 2,09 \text{ s}$$

the fraction of critical damping (damping ratio) is

$$\alpha = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} = \frac{90}{2\sqrt{9000 \times 1000}} = 0,015 = 1,5\%$$

$$\omega_d = \omega_n \sqrt{1 - \alpha^2} = 3,0 \sqrt{1 - 0,015^2} = 3,0 \times 0,999887 = 2,99966 \cong \omega_n$$



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$$f_d = \frac{\omega_d}{2\pi} = \frac{2,99966}{2\pi} = 0,477 \cong f_n$$

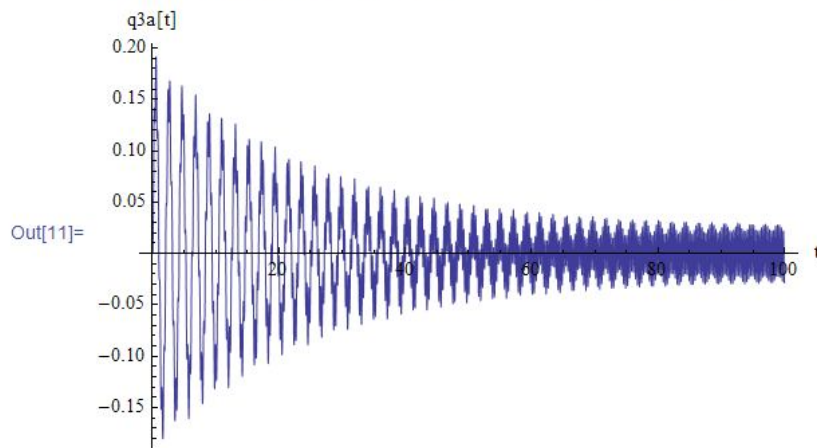
the period of damped free vibrations is

$$T_d = \frac{1}{f_d} = \frac{1}{f_n} = T_n = 2,09s$$

and the period of forced vibrations is

$$T_n = \frac{2\pi}{p} = \frac{2\pi}{20} = 0,314 \text{ s}$$

```
In[11]:= Block[{m = 1000, k = 9000, c = 80, F0 = 10 000, p = 20},  
  Plot[q3a[t], {t, 0, 100}, PlotRange -> All, AxesLabel -> {"t", "q3a[t]"}]]
```

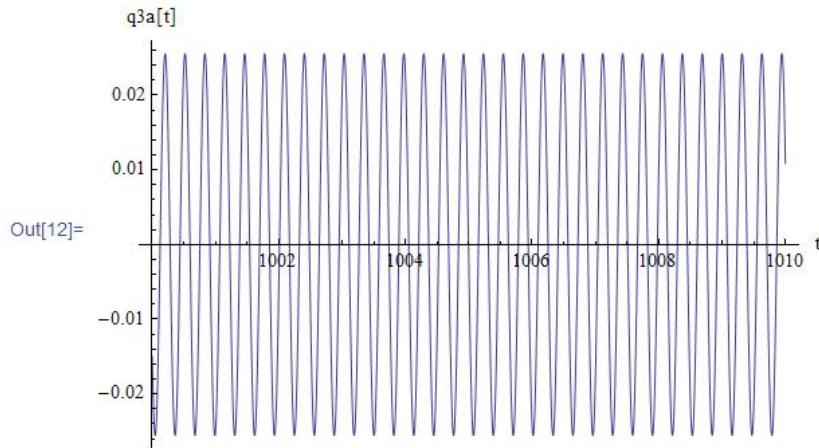


The harmonically forced vibrations of a SDOF system are described by a differential nonhomogenous equation of motion. The solution of such a type of equation is the sum of a general integral and a particular integral. The general integral can be interpreted as free vibration of the system, which occurs with the natural frequency and fades due to damping. The time during which the free vibration persists is called the transient process. The particular integral represents forced vibration with a constant amplitude and a frequency equal to the frequency of the load. The point in time when the general integral is practically equal to zero, and only the particular integral remains, is the beginning of the steady-state process. The figure above shows the transient process, in which we can see the decay of the vibration. These vibrations are a sum of free vibration with the period 2.09s and forced vibration whose period is 0.314s. The motion corresponding to the free vibration is predominant. As the vibration decays, after 1000s, the free vibration nears zero, and only the forced vibration remains. In the next figure we see the steady state vibration only with the period 0.314s.



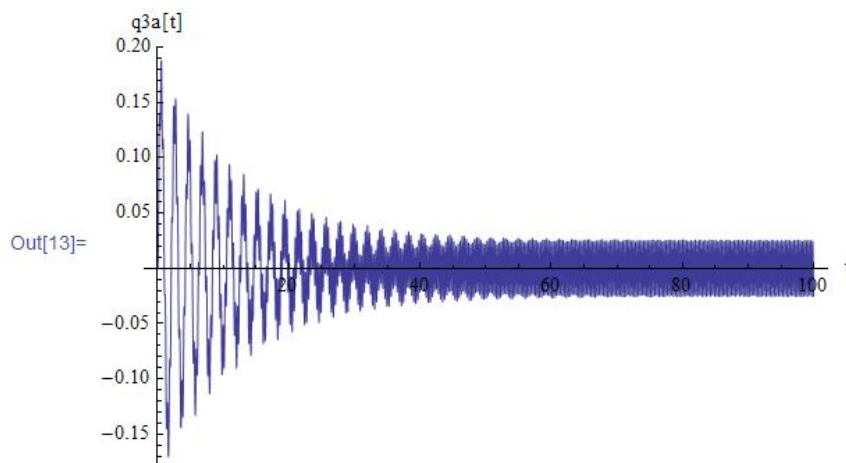
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```
In[12]:= Block[{m = 1000, k = 9000, c = 80, F0 = 10 000, p = 20},  
Plot[q3a[t], {t, 1000, 1010}, PlotRange -> All, AxesLabel -> {"t", "q3a[t"]}]]
```



The greater the damping, the shorter the time during which general integral predominates. In the figure below, the transient process lasts for the first 100 s of vibration only, as the damping coefficient c is doubled. When $c=160$ Ns/m, the transient process lasts approximately 60s, as shown in the figure below.

```
In[13]:= Block[{m = 1000, k = 9000, c = 160, F0 = 10 000, p = 20},  
Plot[q3a[t], {t, 0, 100}, PlotRange -> All, AxesLabel -> {"t", "q3a[t"]}]]
```



To observe only the particular integral, apply the following “*Mathematica*” command

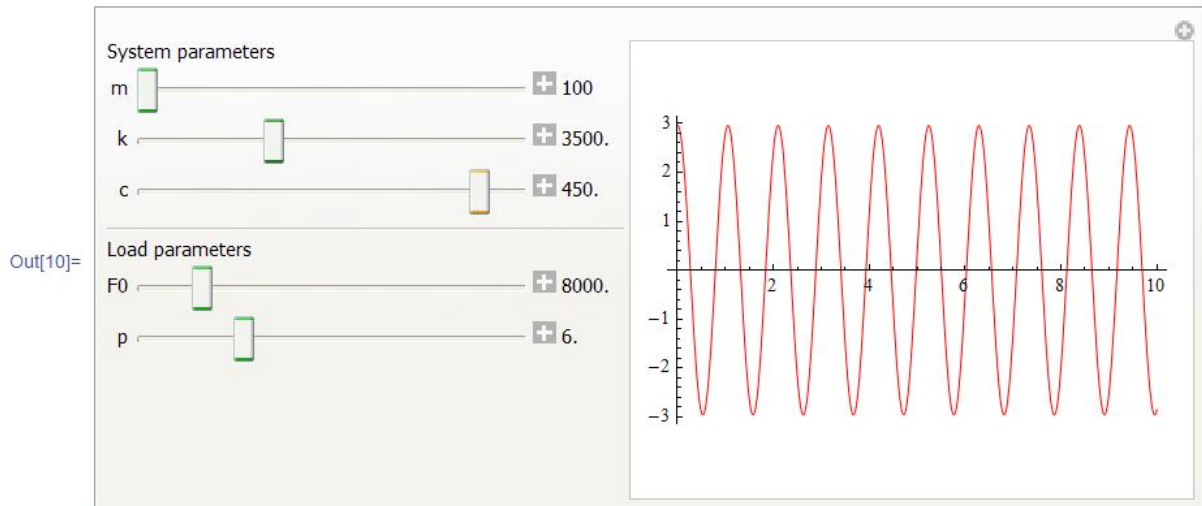
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```

In[10]:= Manipulate[ temp[t_] =  $\frac{F_0}{k \sqrt{\frac{c^2 p^2 + (k - m p^2)^2}{k^2}}} * \text{Sin}[p * t - \text{ArcTan}[\frac{c p}{k - m p^2}]]$ ;

Plot[{temp[t]}, {t, 0, 10}, PlotRange -> All, PlotPoints -> 500, PlotStyle -> {Black, Blue, Red}],
"System parameters", {m, 100, 800, Appearance -> "Labeled"},
{{k, 2000}, 101, 10000, Appearance -> "Labeled"}, {{c, 100}, 0, 500, Appearance -> "Labeled"},
Delimiter, "Load parameters", {{F0, 10000}, 5000, 25000, Appearance -> "Labeled"},
{{p, 10}, 1, 20, Appearance -> "Labeled"}, ControlPlacement -> Left]

```



In engineering applications, the steady-state process is more important. A different form of writing down the above formula may also be used, as presented below.

$$q_p(t) = \frac{v_d}{k} F_s \sin(pt - \psi) + \frac{v_d}{k} F_c \cos(pt - \psi)$$

The amplitude of the dynamic response is equal to

$$\text{am } q_p = \frac{v_d}{k} \sqrt{F_s^2 + F_c^2} = v_d \frac{F_0}{k} = v_d q_{st}$$

which means that the dynamic response is v_d times greater or smaller than the static response q_{st} . v_d is called a dynamic magnification factor

$$v_d = \frac{\text{am } q_p}{q_{st}} = \frac{1}{\sqrt{(1 - \eta^2)^2 + (2\alpha\eta)^2}}$$

$$\psi = \arctan \frac{2\alpha\eta}{1 - \eta^2}$$

$$\eta = \frac{p}{\omega_n} \text{ -- frequency ratio}$$

$$\alpha = \frac{c}{2\sqrt{km}} \text{ -- the fraction of critical damping (damping ratio).}$$



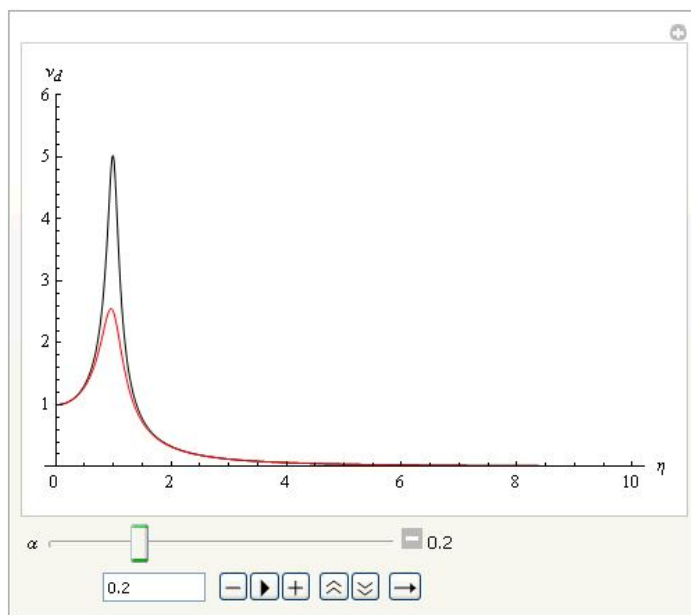
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The following three figures show the diagram of v_d . The first one is drawn on two linear axes, the second one on a vertical logarithmic axis and the last one on two logarithmic axes. The curve with starting parameter $\alpha=0,1$ is drawn in black.

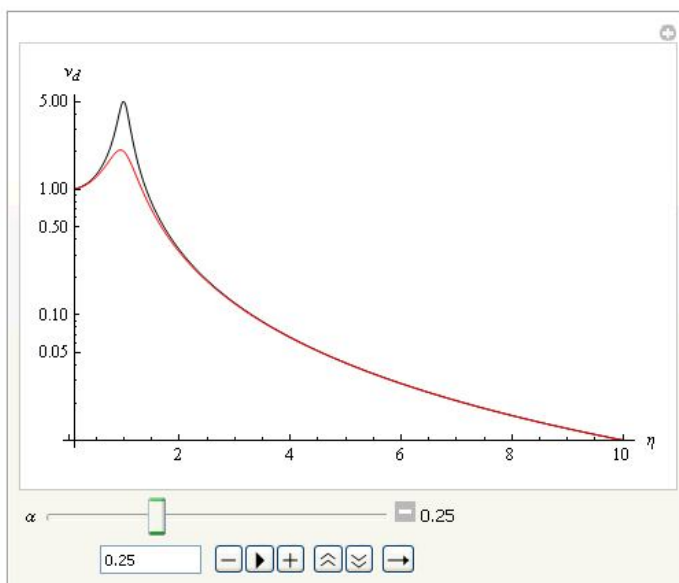
Task:

Find the range of η where the dynamic magnification factor practically does not depend on the critical damping factor.

```
Manipulate[Plot[{1/Sqrt[(1-η^2)^2+(2*0.1*η)^2], 1/Sqrt[(1-η^2)^2+(2*α*η)^2]},
{η, 0, 10}, PlotPoints→500, PlotStyle→{Black, Red}, AxesLabel→{"η", Subscript["v", "d"]},
PlotRange→{0, 6}], {{α, 0.1}, 0, 0.8, Appearance→"Labeled"}, ControlPlacement→Bottom]
```



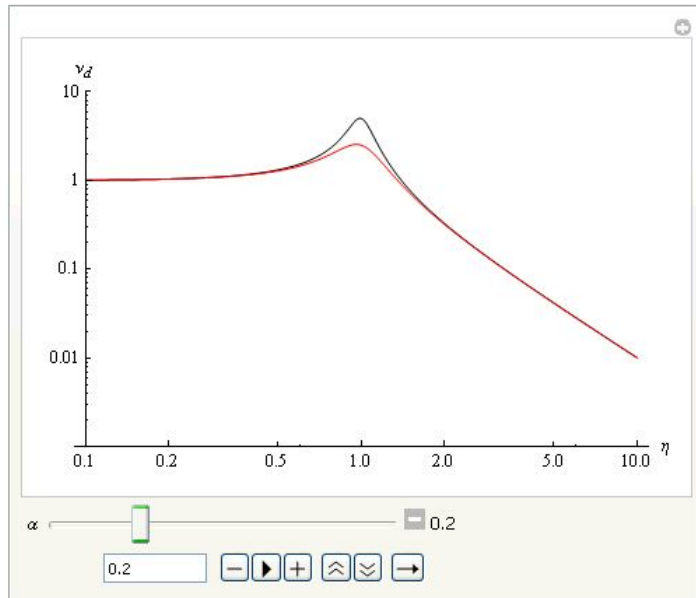
```
Manipulate[LogPlot[{1/Sqrt[(1-η^2)^2+(2*0.1*η)^2], 1/Sqrt[(1-η^2)^2+(2*α*η)^2]},
{η, 0.1, 10}, PlotPoints→500, PlotStyle→{Black, Red}, AxesLabel→{"η", Subscript["v", "d"]},
PlotRange→{0, 10}], {{α, 0.1}, 0, 0.8, Appearance→"Labeled"}, ControlPlacement→Bottom]
```





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```
Manipulate[LogLogPlot[{1/Sqrt[(1-η^2)^2+(2*0.1*η)^2], 1/Sqrt[(1-η^2)^2+(2*α*η)^2]},
{η, 0.1, 10}, PlotPoints → 500, PlotStyle → {Black, Red}, AxesLabel → {"η", Subscript["v", "d"]},
PlotRange → {0.001, 10}], {{α, 0.1}, 0, 0.8, Appearance → "Labeled"}, ControlPlacement → Bottom]
```



The amplitude of response depends not only on the dynamic magnification factor v_d but also on the stiffness of the system k . The formula for $am\ q_p$ could be rewritten in terms of system parameters k , m , c .

$$am\ q_p = v_d \frac{F_o}{k} = \frac{F_o}{k \sqrt{(1-\eta^2)^2 + (2\alpha\eta)^2}}$$

$$\text{substituting } \eta = \frac{p}{\omega_n} \text{ and } \alpha = \frac{c}{2\sqrt{km}}$$

you obtain

```
α := c / 2 / Sqrt[k * m]; η := p / Sqrt[k / m]; v := 1 / Sqrt[(1 - η^2)^2 + (2 * α * η)^2];
ψ := ArcTan[2 * c / 2 / Sqrt[k * m] * η / (1 - η^2)];
```

```
FullSimplify[v * F0 / k, k > 0]
```

$$\frac{F_0}{\sqrt{c^2 p^2 + (k - m p^2)^2}}$$

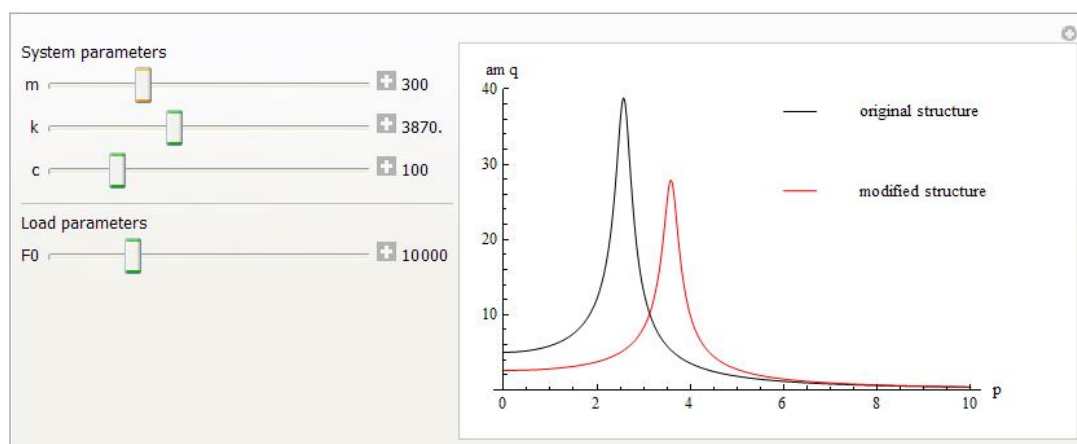
$$am\ q_p = v_d \frac{F_o}{k} = \frac{F_o}{\sqrt{c^2 p^2 + (k - m p^2)^2}}$$



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Using following commands you can observe amplitude of original structure (black line) and modified structure i.e. with changed m , k or c (red line). You can observe that there are region in which response of modified structure decreases but there are also region where the response increases.

```
Manipulate[Plot[{ $\frac{10000}{\sqrt{100^2 p^2 + (2000 - 300 p^2)^2}}$ ,  $\frac{F0}{\sqrt{c^2 p^2 + (k - m p^2)^2}}$ }, {p, 0, 10}, PlotRange -> {0, 40},
PlotPoints -> 500, PlotStyle -> {Black, Red}, AxesLabel -> {"p", "am q"},
PlotLegend -> {"original structure", "modified structure"}, LegendPosition -> {0, 0}, LegendShadow -> None],
"System parameters", {{m, 300}, 100, 800, Appearance -> "Labeled"},
{{k, 2000}, 10, 10000, Appearance -> "Labeled"}, {{c, 100}, 0, 500, Appearance -> "Labeled"}, Delimiter,
"Load parameters", {{F0, 10000}, 5000, 25000, Appearance -> "Labeled"}, ControlPlacement -> Left]
```



Task

Find:

- how does the response change with change of m , c , k , F_0 ,
- what is the best way to suppress the dynamic response,
- regions where the response is greater even if you decrease the maximum response.

Problem

After five periods, the amplitude of the free vibrations changes from A to $0,8A$. Calculate the amplitude after ten periods. After how many periods will the amplitude become smaller than $0,5 A$?

Solution:

In systems with viscous damping, vibrations decays logarithmically. The logarithmic decrement is defined as follows:

$$\vartheta = \frac{1}{n} \ln \frac{q(t)}{q(t + nT_d)} = \alpha \omega_n T_d = \frac{2\pi\alpha}{\sqrt{1 - \alpha^2}}$$

where n is the number of the observed periods. For the considered system, the logarithmic decrement is equal to



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$$\vartheta = \frac{1}{n} \ln \frac{q(t)}{q(t+nT_d)} = \frac{1}{5} \ln \frac{A}{0.8A} = 0.0446.$$

For a lightly damped system, the following formula is true

$$\vartheta = 2\pi\alpha$$

hence

$$\alpha = \vartheta / 2\pi = 0.0446 / 2\pi = 0.007 = 0.7\%.$$

Using the definition of the logarithmic decrement, you can calculate the amplitude after n periods

$$\vartheta = \frac{1}{n} \ln \frac{q(t)}{q(t+nT_d)} \rightarrow q(t+nT_d) = \frac{q(t)}{\exp(n\vartheta)}$$

the amplitude after 10 periods is

$$A_{10} = \frac{A}{\exp(10 \cdot 0.0446)} = 0.64 A.$$

After how many periods will the amplitude become smaller than 0.5 A?

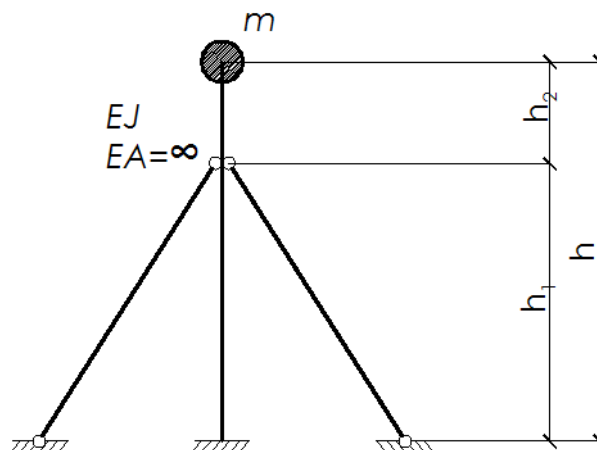
$$\vartheta = \frac{1}{n} \ln \frac{A}{0.5A} \rightarrow n = \frac{1}{\vartheta} \ln \frac{A}{0.5A} = 15.5 \rightarrow n = 16.$$

After the 16th period, the amplitude of motion become smaller than 50% of the initial amplitude

Problem

Find the natural vibrations period of a water tower with a tripod. The tower's capacity for storing water is $V=200 \text{ m}^3$. The axial deformation and the mass of the structure is negligible. How will the natural frequency change in case of the structure without the tripod.

Data: $\rho = 1 \text{ kg/dm}^3$, $E = 200 \text{ GPa}$, $J = 200000 \text{ cm}^4$, $h_1 = 10 \text{ m}$, $h_2 = 5 \text{ m}$.

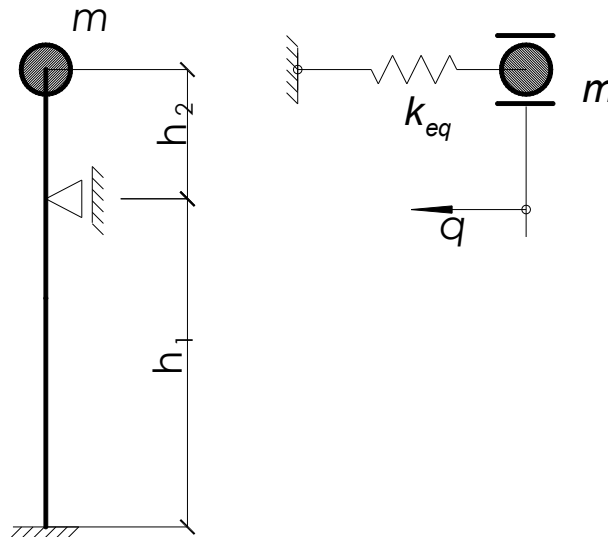




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Solution:

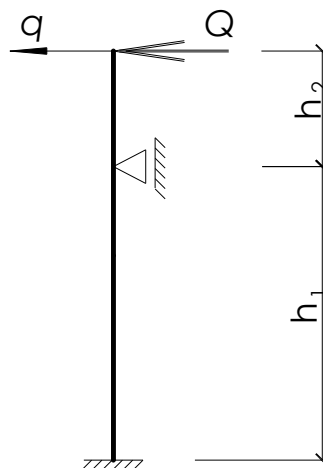
As the axial deformation may be neglected, following scheme could be accepted



As the mass of the structure may be neglected, m is equal to the mass of water, $m = V \times \rho = 200 \times 10^3 \times 1 = 2 \cdot 10^5 \text{ kg}$.

The equivalent stiffness can be calculated from formula

$k = Q/q$, where q – displacement caused by the force Q . This structure is statically indeterminate, so the displacement can be calculated by any method suitable for statically indeterminate structures (flexibility method, displacement method), or with the use engineering tables.



For the above scheme

$$q = \frac{Q h_2^2 (3h_1 + 4h_2)}{12EJ}$$

$$k = \frac{Q}{q} = \frac{12EJ}{h_2^2 (3h_1 + 4h_2)} = \frac{12 \times 200 \cdot 10^9 \times 200000 \cdot 10^{-8}}{5^2 (3 \times 10 + 4 \times 5)} = 3,84 \cdot 10^6 \text{ N/m}$$



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the natural angular frequency is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3,84 \cdot 10^6}{2 \cdot 10^5}} = 4,38 \text{ rad/s}$$

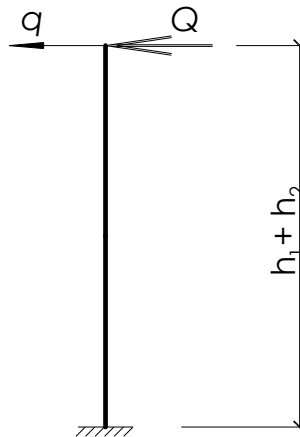
the natural frequency is

$$f_n = \frac{\omega_n}{2\pi} = \frac{4,38}{2\pi} = 0,70 \text{ Hz}$$

and the period of undamped free vibrations is

$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} = 1,43 \text{ s}$$

When a structure without a tripod is considered, the following scheme will be valid



In this case

$$q = \frac{Q(h_1 + h_2)^3}{3EJ}$$

$$k = \frac{Q}{q} = \frac{3EJ}{(h_1 + h_2)^3} = \frac{3 \times 200 \cdot 10^9 \times 200000 \cdot 10^{-8}}{(10 + 5)^3} = 0,356 \cdot 10^6 \text{ N/m}$$

the natural angular frequency is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0,356 \cdot 10^6}{2 \cdot 10^5}} = 1,33 \text{ rad/s}$$

the natural frequency is

$$f_n = \frac{\omega_n}{2\pi} = \frac{1,33}{2\pi} = 0,212 \text{ Hz}$$

and the period of undamped free vibrations is

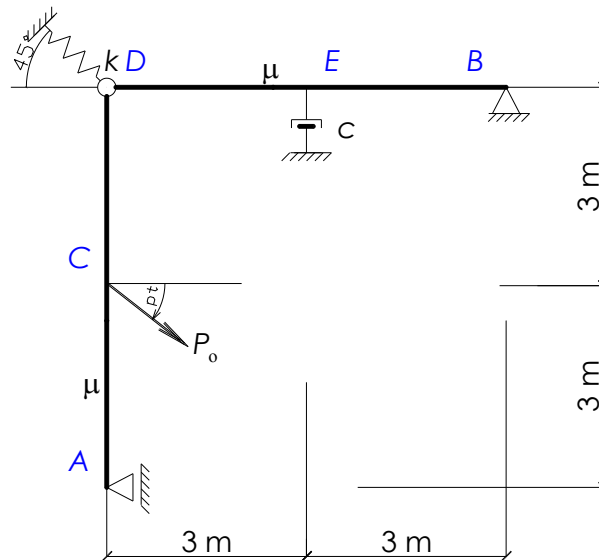
$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} = 4,71 \text{ s}$$

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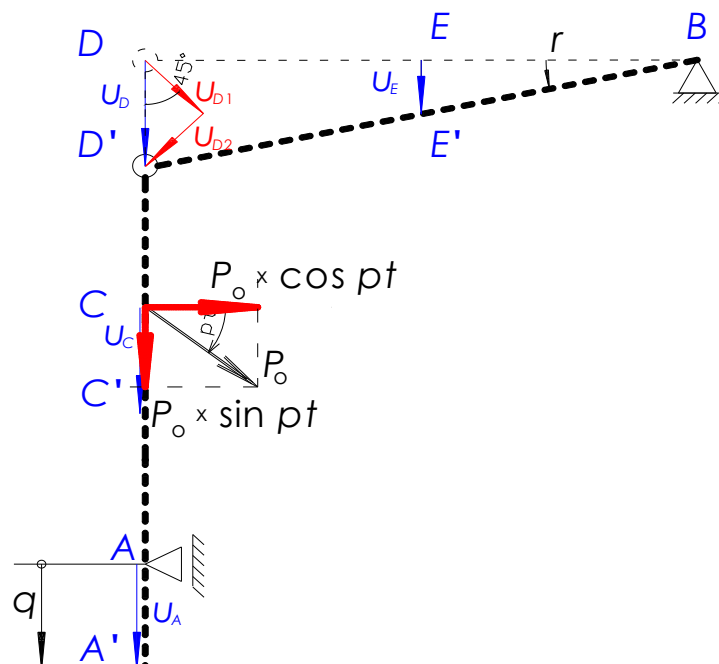
Problem

Choose a generalized coordinate and formulate an equation of motion. Find the parameters of motion and amplitude of vibration in point A. Bars are undeformable.

Data: $\mu = 200 \text{ kg/m}$, $c = 10000 \text{ Ns/m}$, $k = 10^6 \text{ N/m}$, $p = 20 \text{ rad/s}$, $P_0 = 10 \text{ kN}$.



The system consists of two rigid bodies, i.e. non-deformable elements (bar A-D and D-B), and five constraints (two in pin support B, two in hinge D and one in pin-roller support A). As a rigid body on a plane has three degrees of freedom, two bodies have six degrees of freedom. The constraints in this system, however, leave only one degree of freedom. It is therefore necessary to choose a generalized coordinate, which will describe any possible (virtual) displacement in the system, e.g. a vertical displacement of point A (q) or a rotation around support B (coordinate r).



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The figure above shows the scheme of displacement. Every virtual displacement could be defined as the generalized coordinate. The equation of motion will first be written down using coordinate q , and then using coordinate r .

$$u_A = u_C = u_D = q$$

$$\dot{u}_A = \dot{u}_C = \dot{u}_D = \dot{q}$$

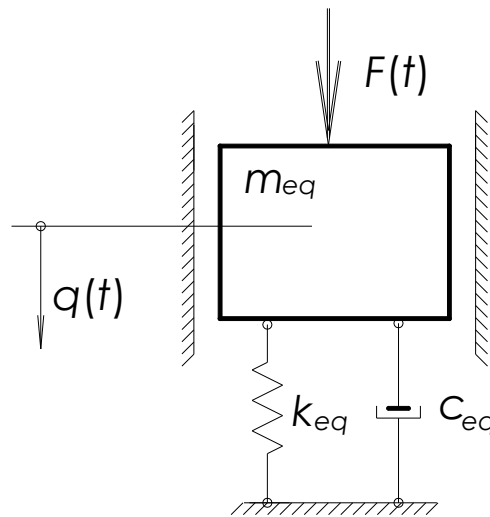
$$u_E = \frac{1}{2} q$$

$$\dot{u}_E = \frac{1}{2} \dot{q}$$

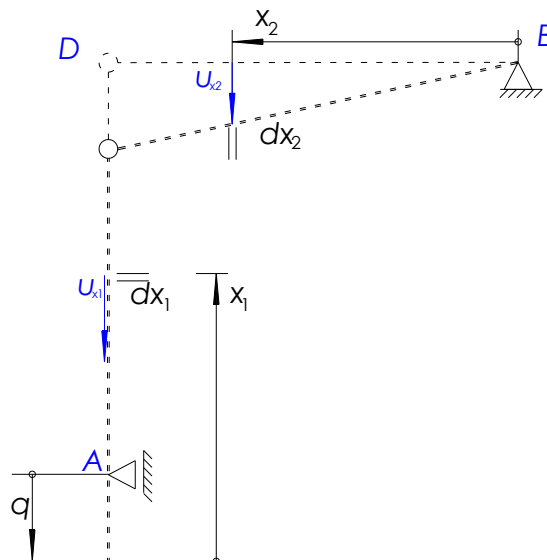
$$u_{D1} = u_D \times \cos 45^\circ = \frac{\sqrt{2}}{2} u_D$$

$$\dot{u}_{D1} = \frac{\sqrt{2}}{2} \dot{q}$$

Expressions for energies will be established both for a real system and for a dynamic model (shown in the figure beneath). The properties of the dynamic model – m_{eq} , c_{eq} , k_{eq} , $F(t)$ – could be calculated from the equality of energies in both systems.



Kinetic energy





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The local displacement or velocity of a very small part of bar could be written down as a function of q or $\dot{q}$

$$\begin{aligned} u_{x1} &= q & \dot{u}_{x1} &= \dot{q} \\ u_{x2} &= q \frac{x_2}{6} & \dot{u}_{x2} &= \dot{q} \frac{x_2}{6} \end{aligned}$$

Kinetic energy of a small part of bar AD and bar DB

$$\begin{aligned} dE_{k,AD} &= \frac{1}{2} dm \dot{u}_{x1}^2 = \frac{1}{2} \mu dx_1 \dot{q}^2 = \frac{1}{2} \mu \dot{q}^2 dx_1 \\ dE_{k,DB} &= \frac{1}{2} dm \dot{u}_{x2}^2 = \frac{1}{2} \mu dx \left(\dot{q} \frac{x_2}{6} \right)^2 = \frac{1}{2} \cdot \frac{x_2^2}{36} \mu \dot{q}^2 dx_2 \end{aligned}$$

The total kinetic energy:

$$\begin{aligned} E_k &= \int_0^{l_{AD}} dE_{k,AD} + \int_0^{l_{DB}} dE_{k,DB} = \int_0^6 \frac{1}{2} \mu \dot{q}^2 dx_1 + \int_0^6 \frac{1}{2} \frac{x_2^2}{36} \mu \dot{q}^2 dx_2 = \\ &= \left[\frac{1}{2} \mu \dot{q}^2 x_1 \right]_0^6 + \left[\frac{1}{2} \frac{x_2^3}{36} \mu \dot{q}^2 \right]_0^6 = \frac{1}{2} \mu \dot{q}^2 6 + \frac{1}{2} \frac{6^3}{36} \mu \dot{q}^2 = \\ &= \frac{1}{2} 6 \mu \dot{q}^2 + \frac{1}{2} 2 \mu \dot{q}^2 = \frac{1}{2} (8 \mu) \dot{q}^2 \end{aligned}$$

The total kinetic energy for the equivalent system is equal to

$$E_k = \frac{1}{2} m_{eq} \dot{q}^2$$

The system is an equivalent one if the energy has the same value as in the real system, so

$$\frac{1}{2} m_{eq} \dot{q}^2 = \frac{1}{2} (8 \mu) \dot{q}^2 \rightarrow m_{eq} = 8 \mu = 1600 \text{ kg}$$

Rayleigh Dissipation Function

The dissipation function is equal to

$$\Theta = \frac{1}{2} c \dot{u}_E^2 = \frac{1}{2} c \left(\frac{1}{2} \dot{q} \right)^2 = \frac{1}{2} c \frac{1}{4} \dot{q}^2 = \frac{1}{2} \left(\frac{1}{4} c \right) \dot{q}^2$$

For the equivalent system, the dissipation function is equal to

$$\Theta = \frac{1}{2} c_{eq} \dot{q}^2$$

The dissipation function should be the same in both formulations

$$\frac{1}{2} c_{eq} \dot{q}^2 = \frac{1}{2} \left(\frac{1}{4} c \right) \dot{q}^2 \rightarrow c_{eq} = \frac{1}{4} c = 2500 \text{ Ns/m}$$

Potential energy

The potential energy for a single spring element is equal to

$$\frac{1}{2} k u_k^2$$



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where u_k is the change in length of the spring element. In the considered system the spring element has one end fixed, so the change in spring length will be equal to that part of the displacement of the other end (point D) which is parallel to the spring axis. The total displacement of point D is u_D , the part of this displacement parallel to the spring axis is u_{D1} , and the part of total displacement perpendicular to the spring axis is u_{D2} . The potential energy for the system is

$$E_p = \frac{1}{2} k u_{D1}^2 = \frac{1}{2} k \left(\frac{\sqrt{2}}{2} q \right)^2 = \frac{1}{2} k \frac{1}{2} q^2 = \frac{1}{2} \left(\frac{1}{2} k \right) q^2$$

The potential energy for the equivalent system is

$$E_p = \frac{1}{2} k_{eq} q^2$$

From the equality of these formulas

$$\frac{1}{2} k_{eq} q^2 = \frac{1}{2} \left(\frac{1}{2} k \right) q^2 \rightarrow k_{eq} = \frac{1}{2} k = 5 \cdot 10^5 \text{ N/m}$$

Work of acting forces

You will obtain the same results if you consider the vertical and horizontal components of the rotating force P_0 instead of the whole force. The vertical component of the force is $P_0 \sin pt$ and the horizontal component is $P_0 \cos pt$. The work of the acting force is equal to

$$\begin{aligned} L &= P_0 \cos pt \times u_{C, horizontal} + P_0 \sin pt \times u_{C, vertical} = \\ &= P_0 \cos pt \times 0 + P_0 \sin pt \times u_C = P_0 \sin pt \times u_C = P_0 \sin pt \times q \end{aligned}$$

The work of the acting force for the equivalent system is

$$L = F(t)q$$

hence the equivalent dynamic force is

$$F(t) = P_0 \sin pt = 10000 \sin (20 t) \text{ [N]}$$

Equation of motion

$$m_{eq} \ddot{q}(t) + c_{eq} \dot{q}(t) + k_{eq} q(t) = F(t)$$

Substituting the obtained values

$$8\mu \ddot{q}(t) + \frac{1}{4} c \dot{q}(t) + \frac{1}{2} k q(t) = P_0 \sin pt$$

$$1600 \ddot{q}(t) + 2500 \dot{q}(t) + 5 \cdot 10^5 q(t) = 10000 \sin 20t$$

Parameters of motion

the natural angular frequency is

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{5 \cdot 10^5}{1600}} = 17,68 \text{ rad/s}$$

the natural frequency is



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$$f_n = \frac{\omega_n}{2\pi} = \frac{17,68}{2\pi} = 2,81 \text{ Hz}$$

the period of undamped free vibration is

$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} = 0,355 \text{ s}$$

the fraction of critical damping (damping ratio) is

$$\alpha = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} = \frac{2500}{2\sqrt{5 \cdot 10^5 \times 1600}} = 0,044 = 4,4\%$$

$$\omega_d = \omega_n \sqrt{1 - \alpha^2} = 17,68 \sqrt{1 - 0,044^2} = 17,68 \times 0,9990 = 17,66 \text{ rad/s}$$

$$f_d = \frac{\omega_d}{2\pi} = \frac{17,66}{2\pi} = 2,81 \text{ Hz}$$

the period of damped free vibrations is

$$T_d = \frac{1}{f_d} = \frac{2\pi}{\omega_d} = 0,356 \text{ s}$$

and the logarithmic decrement of damping is

$$\vartheta = \alpha \omega_n T_d = \frac{2\pi\alpha}{\sqrt{1 - \alpha^2}} = 0,277$$

Steady-state solution of equation of motion

The equation of motion for forced vibration is

$$m_{eq}\ddot{q} + c_{eq}\dot{q} + k_{eq}q = F(t) = F_s \sin pt + F_c \cos pt$$

The steady-state (particular) solution of this equation is

$$q_p(t) = q_s \sin pt + q_c \cos pt$$

$$q_p(t) = \frac{v_d}{k_{eq}} F_s \sin(pt - \psi) + \frac{v_d}{k_{eq}} F_c \cos(pt - \psi)$$

the amplitude of this motion is equal to

$$\text{am } q_p = \frac{v_d}{k_{eq}} \sqrt{F_s^2 + F_c^2} = v_d \frac{F_o}{k_{eq}} = v_d q_{st}$$

where the dynamic magnification factor

$$v_d = \frac{1}{\sqrt{(1 - \eta^2)^2 + (2\alpha\eta)^2}}, \quad \eta = \frac{p}{\omega_n} = \frac{20}{17,68} = 1,13$$



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$$\nu_d = \frac{1}{\sqrt{(1-1,13^2)^2 + (2 \times 0,044 \times 1,13)^2}} = 3,40 \quad \text{which means that the response of the system is over}$$

three times greater than the response of the same system when the same force is applied statically.

In this problem $F_s = P_0$, $F_c = 0$, and the amplitude of motion is equal to

$$\text{am } q_p = \nu_d \frac{F_s}{k_{eq}} = \nu_d \frac{P_0}{k_{eq}} = 3,40 \frac{10 \cdot 10^3}{5 \cdot 10^5} = 3,40 \times 0,02 = 0,068 \text{ m}$$

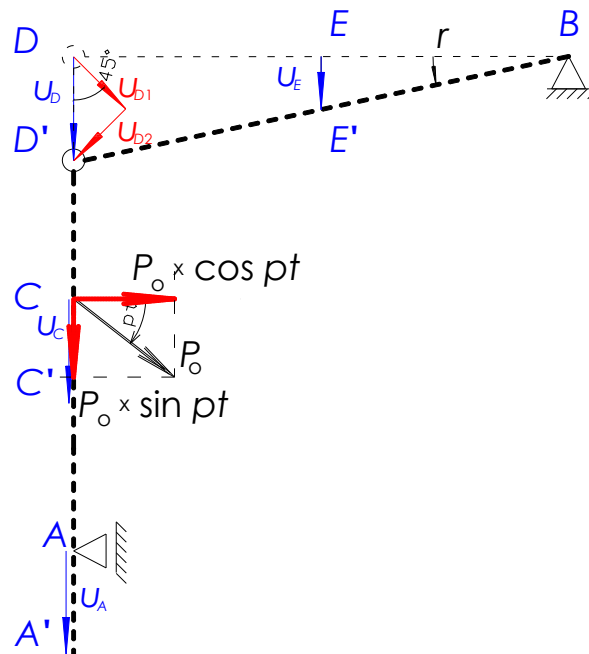
The amplitude of vibration of point A is equal to the amplitude of q_p , am $u_A = 0,068 \text{ m}$.

Problem

Solve the previous problem choosing a different generalized coordinate.

Solution

This time let us choose the rotation r around support B as the dynamic coordinate.



The scheme of displacement is shown in the figure above. Every possible displacement could be defined by the generalized coordinate r .

$$u_A = u_C = u_D = r \times 6m$$

$$\dot{u}_A = \dot{u}_C = \dot{u}_D = 6\dot{r}$$

$$u_E = r \times 3m$$

$$\dot{u}_E = 3\dot{r}$$

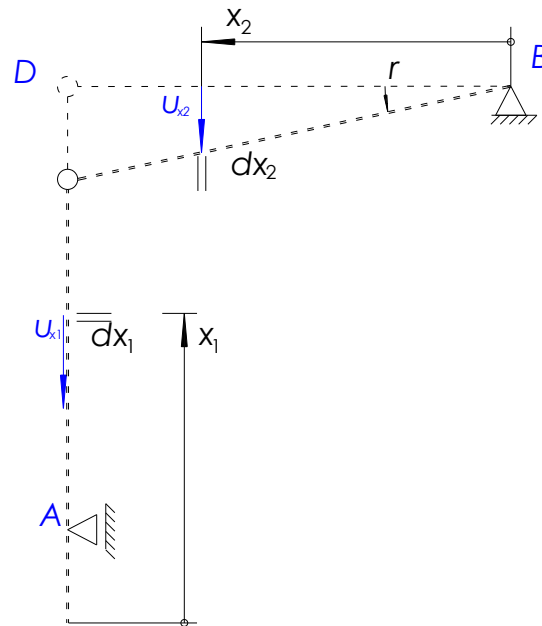


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$$U_{D1} = U_D \times \cos 45^\circ = \sqrt{2}/2 U_D$$

$$\dot{U}_{D1} = 3\sqrt{2} \dot{r}$$

Kinetic energy



The local displacement and velocity could be written down as a function of r and $\dot{r}$

$$\begin{aligned} U_{x1} &= 6r & \dot{U}_{x1} &= 6\dot{r} \\ U_{x2} &= r x_2 & \dot{U}_{x2} &= \dot{r} x_2 \end{aligned}$$

The kinetic energy of a small part of bar AD and bar DB is

$$\begin{aligned} dE_{k,AD} &= \frac{1}{2} dm \dot{U}_{x1}^2 = \frac{1}{2} \mu dx_1 36 \dot{r}^2 = \frac{1}{2} 36 \mu \dot{r}^2 dx_1 \\ dE_{k,DB} &= \frac{1}{2} dm \dot{U}_{x2}^2 = \frac{1}{2} \mu dx_2 (\dot{r} x_2)^2 = \frac{1}{2} \cdot x_2^2 \mu \dot{r}^2 dx_2 \end{aligned}$$

The total kinetic energy is

$$\begin{aligned} E_k &= \int_0^{AD} dE_{k,AD} + \int_0^{DB} dE_{k,DB} = \int_0^6 \frac{1}{2} 36 \mu \dot{r}^2 dx_1 + \int_0^6 \frac{1}{2} x_2^2 \mu \dot{r}^2 dx_2 = \\ &= \left[\frac{1}{2} 36 \mu \dot{r}^2 x_1 \right]_0^6 + \left[\frac{1}{2} \frac{1}{3} x_2^3 \mu \dot{r}^2 \right]_0^6 = \frac{1}{2} \mu \dot{r}^2 36 \times 6 + \frac{1}{2} \frac{1}{3} 6^3 \mu \dot{r}^2 = \\ &= \frac{1}{2} 216 \mu \dot{r}^2 + \frac{1}{2} 72 \mu \dot{r}^2 = \frac{1}{2} (288 \mu) \dot{r}^2 \end{aligned}$$



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The total kinetic energy for the equivalent system is equal to

$$E_k = \frac{1}{2} m_{eq} \dot{r}^2$$

A system is equivalent if the energy has the same value as in the real system, so

$$\frac{1}{2} m_{eq} \dot{r}^2 = \frac{1}{2} (288\mu) \dot{r}^2 \rightarrow m_{eq} = 288\mu = 57600 \text{ kg} \cdot \text{m}^2 / \text{rad} [\text{kg m}^2]$$

Rayleigh Dissipation Function

The dissipation function is equal to

$$\Theta = \frac{1}{2} C \dot{U}_E^2 = \frac{1}{2} C (3\dot{r})^2 = \frac{1}{2} C 9\dot{r}^2 = \frac{1}{2} (9C) \dot{r}^2$$

For the equivalent system, the dissipation function is equal to

$$\Theta = \frac{1}{2} C_{eq} \dot{r}^2$$

The dissipation function should be the same in both formulations

$$\frac{1}{2} C_{eq} \dot{r}^2 = \frac{1}{2} (9C) \dot{r}^2 \rightarrow C_{eq} = 9C = 90000 \text{ N m s/rad} [\text{N m s}]$$

Potential energy

Applying the same method as in the previous solution

$$E_p = \frac{1}{2} k U_{D1}^2 = \frac{1}{2} k \left(\frac{\sqrt{2}}{2} 6r \right)^2 = \frac{1}{2} k 18r^2 = \frac{1}{2} (18k) r^2$$

Potential energy for the equivalent system

$$E_p = \frac{1}{2} k_{eq} r^2$$

From the equality of these formulas

$$\frac{1}{2} k_{eq} r^2 = \frac{1}{2} (18k) r^2 \rightarrow k_{eq} = 18k = 18 \cdot 10^6 \text{ Nm/rad} [\text{Nm}]$$

Work of acting forces

The work of the acting force is equal to

$$\begin{aligned} L &= P_0 \cos pt \times u_{C, \text{horizontal}} + P_0 \sin pt \times u_{C, \text{vertical}} = \\ &= P_0 \cos pt \times 0 + P_0 \sin pt \times u_C = P_0 \sin pt \times u_C = P_0 \sin pt \times 6r = 6 P_0 \sin pt \times r \end{aligned}$$

The work of the acting force for the equivalent system is

$$L = F(t)q$$

hence the equivalent dynamic force is

$$F(t) = 6 P_0 \sin pt = 60000 \sin (20 t) [\text{Nm}]$$

Equation of motion



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$$m_{eq} \ddot{q}(t) + c_{eq} \dot{q}(t) + k_{eq} q(t) = F(t)$$

After substituting the evaluated values, we achieve

$$288\mu \ddot{q}(t) + 9c \dot{q}(t) + 18k q(t) = 6P_0 \sin pt$$

$$57600 \ddot{q}(t) + 90000 \dot{q}(t) + 18 \cdot 10^6 q(t) = 60000 \sin 20t$$

Parameters of motion

The natural angular frequency is

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{18 \cdot 10^6}{57600}} = 17,68 \text{ rad/s}$$

the natural frequency is

$$f_n = \frac{\omega_n}{2\pi} = \frac{17,68}{2\pi} = 2,81 \text{ Hz}$$

the period of undamped free vibrations is

$$T_n = \frac{1}{f_n} = \frac{2\pi}{\omega_n} = 0,355 \text{ s}$$

the fraction of critical damping (damping ratio) is

$$\alpha = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} = \frac{90000}{2\sqrt{18 \cdot 10^6 \times 57600}} = 0,044 = 4,4\%$$

$$\omega_d = \omega_n \sqrt{1 - \alpha^2} = 17,68 \sqrt{1 - 0,044^2} = 17,68 \times 0,9990 = 17,66 \text{ rad/s}$$

$$f_d = \frac{\omega_d}{2\pi} = \frac{17,66}{2\pi} = 2,81 \text{ Hz}$$

the period of damped free vibrations is

$$T_d = \frac{1}{f_d} = \frac{2\pi}{\omega_d} = 0,356 \text{ s}$$

and the logarithmic decrement of damping is

$$\vartheta = \alpha \omega_n T_d = \frac{2\pi\alpha}{\sqrt{1 - \alpha^2}} = 0,277.$$

Conclusions:

As in this equation the generalized coordinate is assumed to be a unit angular displacement, the parameters m_{eq} , c_{eq} , k_{eq} , $F(t)$ of the equation are different than the ones for the equation obtained with the use of unit translational displacement (shown above).

Solution of equation of motion



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The amplitude of steady-state motion is equal to

$$\text{am } q_p = \frac{v_d}{k_{eq}} \sqrt{F_s^2 + F_c^2} = v_d \frac{F_o}{k_{eq}} = v_d q_{st}$$

where the dynamic magnification factor

$$v_d = \frac{1}{\sqrt{(1-\eta^2)^2 + (2\alpha\eta)^2}}, \quad \eta = \frac{p}{\omega_n} = \frac{20}{17,68} = 1,13$$

$$v_d = \frac{1}{\sqrt{(1-1,13^2)^2 + (2 \times 0,044 \times 1,13)^2}} = 3,40$$

In this problem, $F_s = P_0$, $F_c = 0$, and the amplitude of motion is equal to

$$\text{am } r_p = v_d \frac{F_s}{k_{eq}} = v_d \frac{P_0}{k_{eq}} = 3,40 \frac{60 \cdot 10^3}{18 \cdot 10^6} = 3,40 \times 3,33 \cdot 10^{-3} = 0,0113 \text{ rad}$$

The amplitude of vibration of point A is equal to

$$u_A = 6r \rightarrow \text{am } u_A = 6 \text{ am } r_p = 6 \times 0,0113 = 0,068 \text{ m}$$

Conclusion: the results do not depend on the choice of the coordinate.